Number fields with Hopf-Galois module structure repeating periodically

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A field extension L/K is Hopf-Galois if L receives a linear action of some K-Hopf algebra H with the same properties that the Galois group algebra $F[G]$ acts on a Galois extension E/F with group G. Such a Hopf algebra together with the linear action is what we call a Hopf-Galois structure on L/K. We also say that L/K is H-Galois. If now L/K is an H-Galois extension of number fields, the relevant question in Hopf-Galois module theory is whether the ring of integers $\mathcal{O}_L$ is free as a module over the associated order $\mathfrak{A}_H$ in H, defined as the set of elements in H whose action on L leaves $\mathcal{O}_L$ invariant. In [1], Rio and I introduced an effective method to answer this question, based on the knowledge of an integral basis of L/K and the action of H on that integral basis. In this talk I shall present an ongoing project on how to apply the aforementioned method to number fields with integral bases with the same coefficients in order to obtain the same Hopf-Galois module structure, and how to use it on number fields with an integral basis repeating periodically. As defined in [3], a parametric family $\{L_m^{(n)}\}_{m\in\mathbb{Z}}$ of degree n number fields has an integral basis repeating periodically if, modulo some integer n_0, L_m and L_{m+kn_0} have integral bases with the same coefficients for every $k \in \mathbb{Z}$. If time allows, we shall see some concrete results on the families of pure number fields or simplest number fields [2].

References

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[3] L. Remete. "Integral basis of pure fields with square-free parameter". In: *Studia Scent. Math. Hungar.* 57 [2020], pp. 91–115.